Entropy and Computational Power of the Universe

Nadie Yiluo LiTenn
Dec 27, 2020
UCSB SPS Journal Club

Warning:

Dimension analysis is present.

Might trigger math police.

Not Rigorous!

$k = 1$

For the sake of dimension analysis

Just conversion factor

Not as fundamental as c, G, h

…

Outline

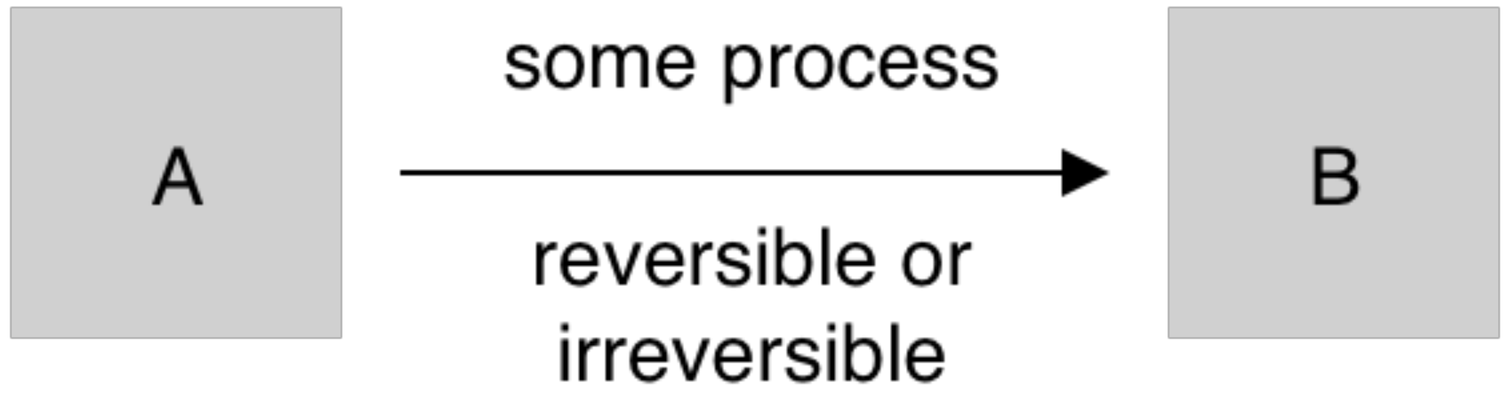
- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

Outline

- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

Motivation for Entropy

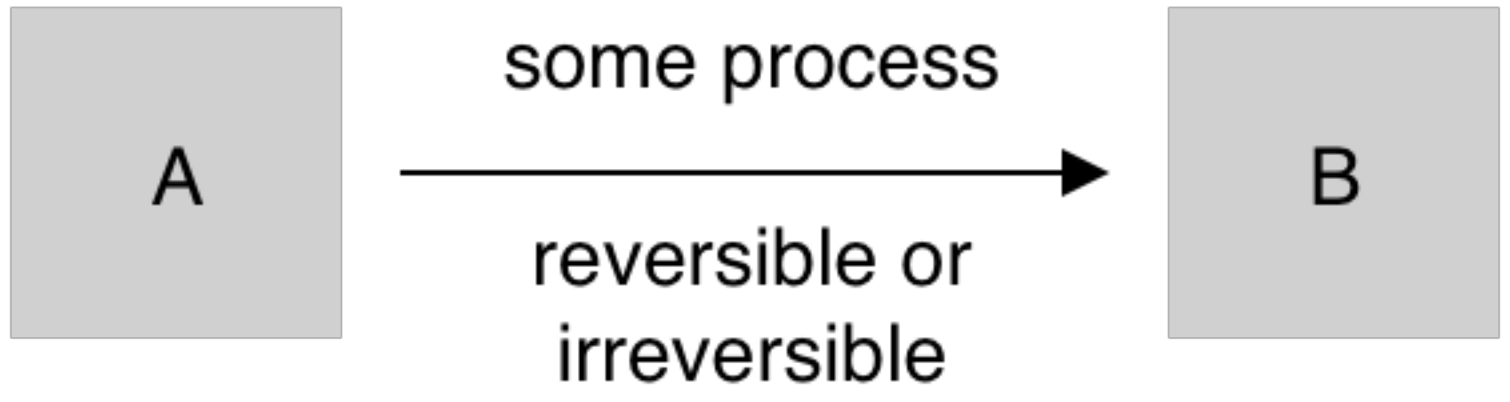
- Consider arbitrary from deterministic initial state **A** to final state **B**



- Is there a quantity that could measure the nature's preference for every state concerned at the moment?
- Remain unchanged when the system considered is undergoing reversible transformation; increase monotonically when undergoing irreversible transformation

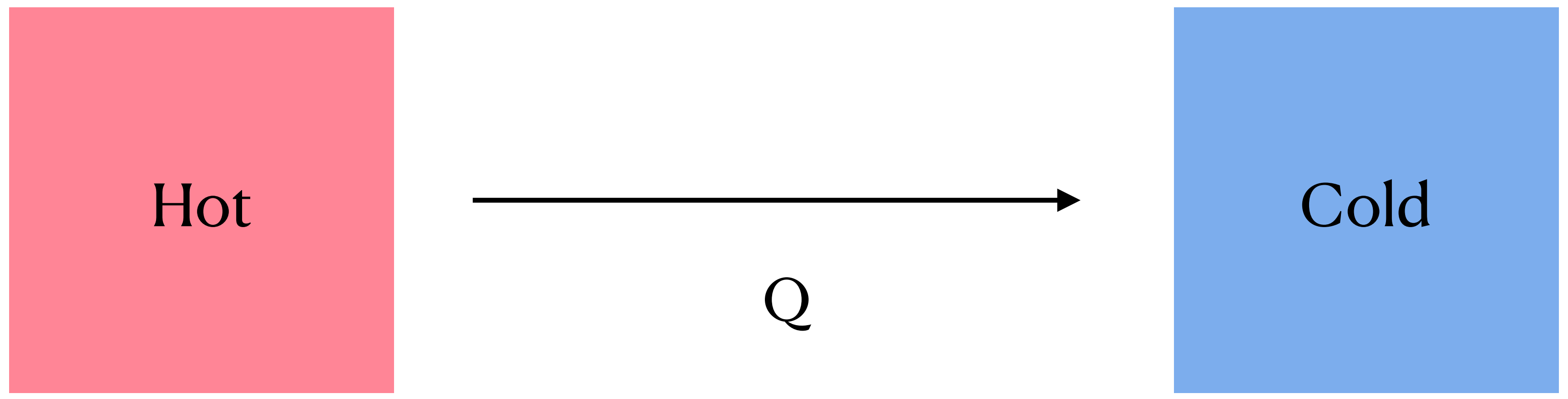
Motivation for Entropy

- Consider arbitrary from deterministic initial state **A** to final state **B**



- Clausius found it: entropy

Motivation for Entropy



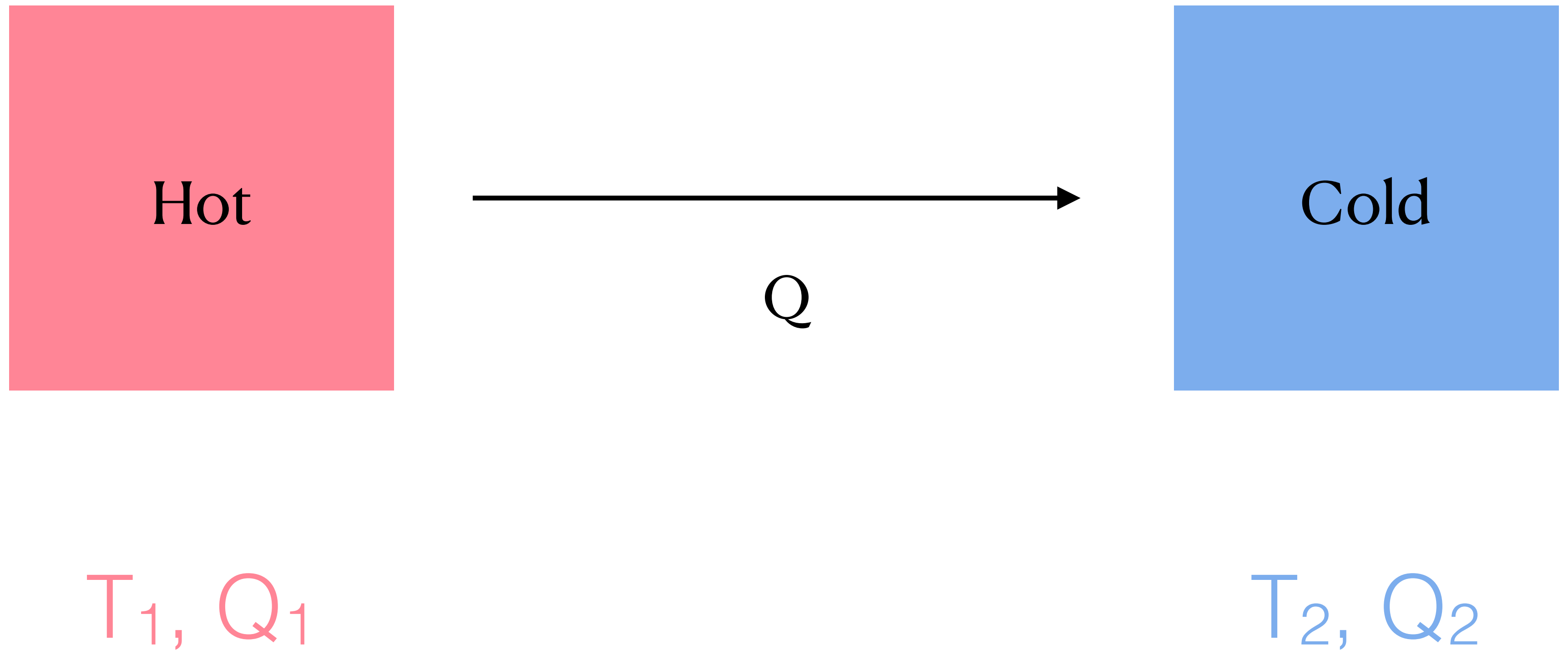
T_1, Q_1 T_2, Q_2

$[T] = [Q] =$ energy

T/Q ? No. When Q = 0, this blows up.

Q/T ? Yes.

Motivation for Entropy

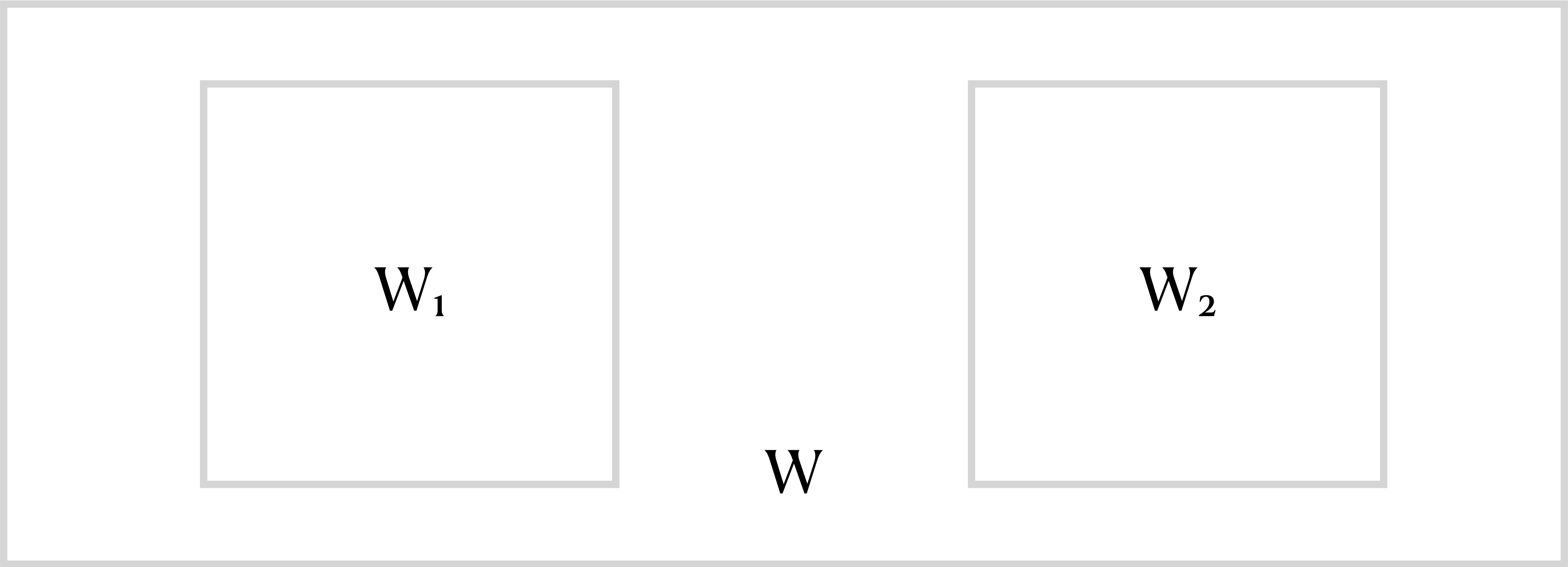


$$\Delta S = -\frac{Q}{T_1} + \frac{Q}{T_2} \geq 0 \quad \longrightarrow \quad \sum_i \frac{Q_i}{T_i} \leq 0$$

Outline

- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

Entropy and Planck



- The total probability is $W = W_1 W_2$
- We speculate that entropy is some function of the probability, and we add the entropy of the two subsystem to get total entropy

$$f(W) = f(W_1 W_2) = f(W_1) + f(W_2)$$

Entropy and Planck

- The total probability is $W = W_1 W_2$
- We speculate that entropy is some function of the probability, and we add the entropy of the two subsystem to get total entropy

$$f(W) = f(W_1 W_2) = f(W_1) + f(W_2)$$

- Take partial derivatives

$$f''(W_1 W_2) W_1 W_2 + f'(W_1 W_2) = \cancelto{0}{f''(W_1)\frac{\partial W_1}{\partial W_2} + f''(W_2)\frac{\partial W_2}{\partial W_1}}$$

$$f''(W)W + f'(W) = 0$$

- General solution is

$$f(W) = c_1 \ln W + c_2$$

Entropy and Planck

- General solution is

$$f(W) = c_1 \ln W + c_2$$

- Planck figured out c_1 to be the Boltzmann constant (since Boltzmann was his idol)
- But just conversion factor
- If we are more logical and let the system have 0 entropy when it is completely classical (i.e. W = 1), and without loss of generality let $c_2 = 0$, then we can completely ditch the Boltzmann constant.
- Connected thermal and statistical

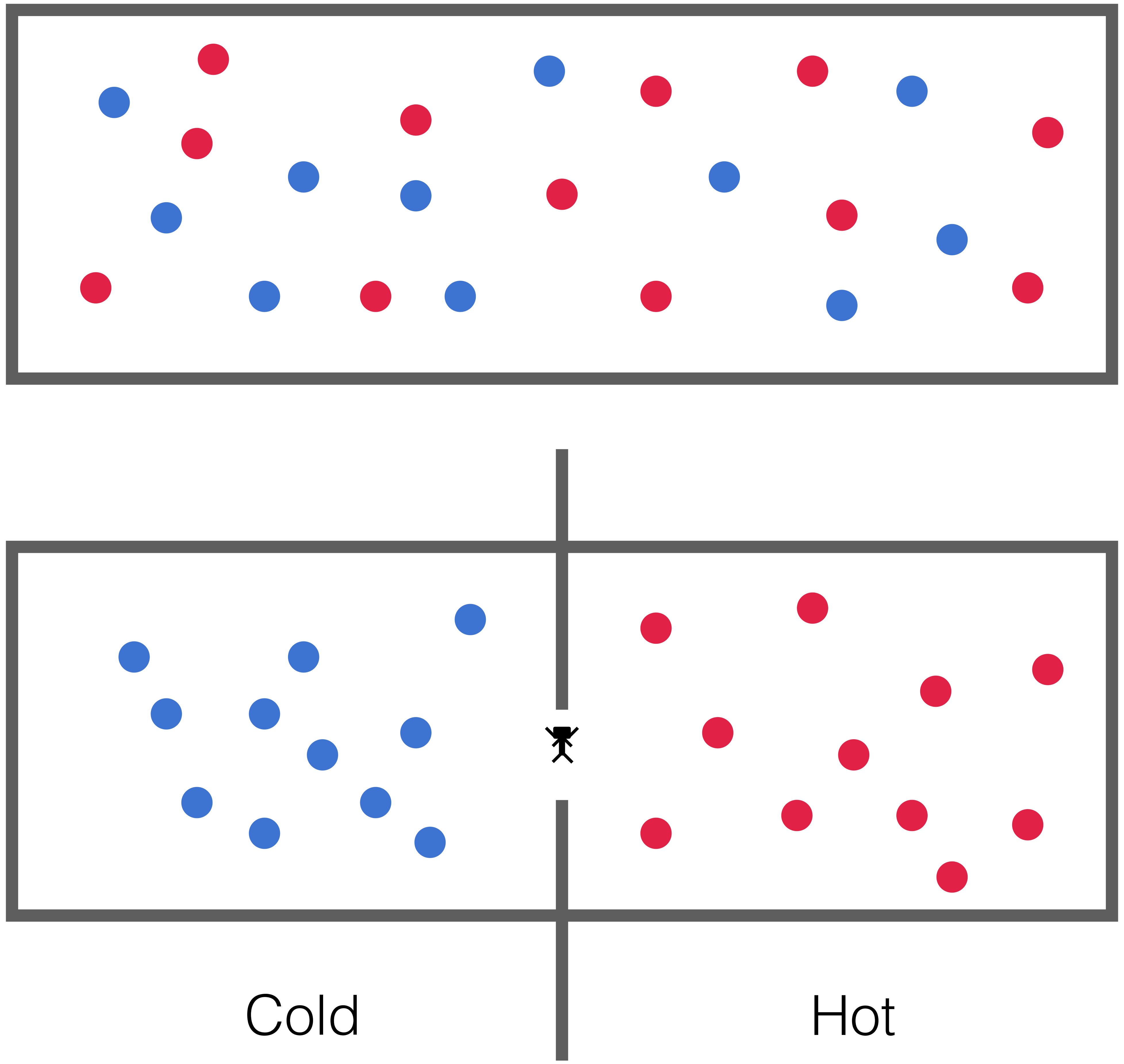
$$f(W) = S = \ln W$$

Outline

- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

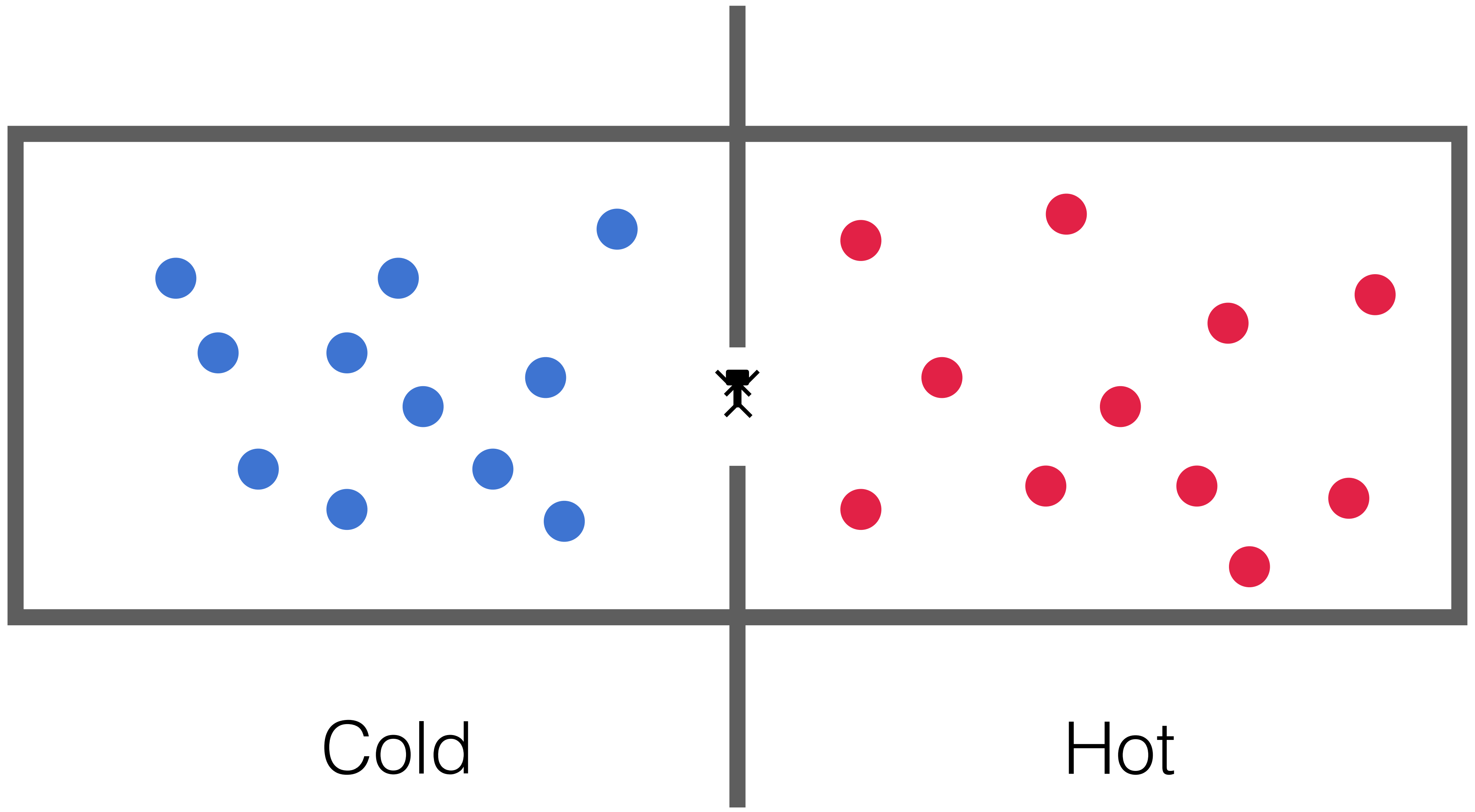
Entropy and Quantum Computing

- Maxwell's Demon



Entropy and Quantum Computing

- Resolution is in the entropy
- Demon is a finite physical system —> must have a finite memory (finite arrangements to store the information of the particles)
- Suppose demon is composed of *n* two-level system —> can have at most 2^n arrangements (*n* bits)
- At some point, the demon has to forget information in order to register new information



$$S = \ln 2$$

$$\Delta E = T \Delta S$$

Paid the energy bill!

Entropy and Quantum Computing

$$S = \ln 2$$

$$\Delta E = T \Delta S$$

- Minimum energy required to erase information: Landauer's Principle
- AND, OR gate in classical computation (2 to 1); must erase 1 bit in each operation
- After n operations, energy cost is $\Delta E \sim nT \ln 2$
- What about reversible computation? Quantum computing is the answer. Everything is unitary.

No energy bill to pay!!

Outline

- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

Entropy and Black Hole

- For a black hole, the entropy is proportional to area

$$S \sim \frac{A}{l_p^2}$$

where $A \sim (\frac{GM}{c^2})^2$. Recall that Schwarzschild radios is $R \sim \frac{GM}{c^2}$

and Planck length $l_p^2 \sim \frac{G\hbar}{c^3}$

Entropy and Black Hole

- For a black hole, the entropy is $S \sim \dfrac{A}{l_p^2}$

- Introduce generalized entropy $S_{gen} = S_{BH} + S_{out}$

  where S_{out} is the entropy of matter going out of the black hole

- Insist on this to get the generalized second law $dS_{gen} \geq 0$

- Sum of Black hole entropy and the ordinary entropy of stuffs in the black hole exterior region never decreases

Entropy and Black Hole

$$S \sim \frac{A}{l_p^2} \qquad\qquad A \sim \left(\frac{GM}{c^2}\right)^2$$

- Back to natural units $c = \hbar = G = k = 1$
- Vary the area of black hole $\delta A \sim M\delta M$
- And by dimension analysis $\delta M \sim \frac{ER}{l}$

$$\delta S_{BH} = \frac{dS_{BH}}{dM}\delta M \sim \frac{\cancelto{1}{M}}{l}ER$$

- Concerned about entropy of arbitrary things outside the black hole, so mass of the black hole is not exactly relevant in the proceeding discussion
- Bekenstein bound:

$$\delta S_{BH} \lesssim ER$$

Proof in QFT: (Casini 2008)

Entropy and Black Hole

- Apply generalized second law with the knowledge that $S_{out} = -S_{matter}$

$$S_{gen} = S_{BH} + S_{out} \qquad\qquad dS_{gen} \geq 0$$

- We get upper bound for entropy of matter

$$dS_{gen} = \delta S_{BH} - S_{matter} \geq 0$$

$$S_{matter} \leq \delta S_{BH} \lesssim ER = \frac{ER}{\hbar c}$$

- Black hole will saturated the upper bound of entropy
- We are ready to talk about computational power

Computational Power

- There are two limits imposed by physics:
  - Energy limits the speed
  - Entropy limits the memory accessible for computation
- Energy:
  - Maximally how many operations can “a matter” perform in unit time?
  - Ask our old friend Heisenberg

$$\Delta x \Delta p \sim \hbar \longrightarrow \Delta E \Delta t \sim \hbar$$

$$\frac{1}{t} \sim \frac{E}{\hbar}$$

Computational Power

- There are two limits imposed by physics:
  - Energy limits the speed $\frac{1}{t} \sim \frac{E}{\hbar}$
  - Entropy limits the memory accessible for computation
- Entropy:
  - Information are stored in bits $I = \log_2 N$ where N is the total number of accessible states
  - In terms of entropy $S = \ln W$ where W is the number of physically accessible states (multiplicity)
  - All computational units are essentially physical matter

$$I = \frac{S}{\ln 2}$$

Computational Power

- There are two limits imposed by physics:
  - Energy limits the speed $\frac{1}{t} \sim \frac{E}{\hbar}$
  - Entropy limits the memory accessible for computation $I = \frac{S}{\ln 2}$
- Merge the two: number of operations per second per bit a system can perform is $\frac{E}{\hbar} \frac{\ln 2}{S}$
- Now hold volume constant, and rewrite above $\frac{T \ln 2}{\hbar}$

- Temperature of the system determines the speed of operations on each bit

Computing with Black Hole

- Can a black hole be "programmed"?
- Hawking & Bekenstein: black hole has entropy, and that information is not destroyed when it falls into the black hole, and will be re-emitted in the form of Hawking radiation. We just need to know how to decode it.
- Then dimension analysis can take care of the rest
    - Hawking radiation: $T_H \sim \frac{\hbar c^3}{GM}$
    - Lifetime of black hole: $t_{BH} \sim \frac{G^2 M^3}{\hbar c^4}$
- Plug and Chug (using $E \sim Mc^2$)
    - A black hole can perform $\sim \frac{Mc^2}{\hbar}$ operations per unit time
    - It can register $\sim \frac{GM^2}{\hbar c} \frac{1}{\ln 2}$ bits in total

Computing with Black Hole

- 1 kg black hole
  - ~ 10^{34} operations per second
  - ~ 10^{15} bits registered in total
- Note that this number of bits is something we can achieve already, but our corresponding computational power is far from comparable to an ultimate computer like black hole

Outline

- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

Ultimate Computational Power of the Universe

- Age of universe is estimated to be: ~10^{10} years ~10^{17} seconds
- The volume (particle horizon and the boundary of all the information we can attain) is $\sim (ct)^3$
  - Total number of operations performed by the universe is $\sim \frac{(ct)^3 \rho c^2}{\hbar} = \frac{\rho c^5 t^4}{\hbar}$

Ultimate Computational Power of the Universe

- Rest of the work is to find density of the universe
  - Must exist some critical density such that
    - Go below, then kinetic energy wins; universe will expand forever
    - Go beyond, then gravitational energy wins; universe will contract
- For galaxies at distance R, it is moving away at speed ~ *HR*, where $H \sim \frac{1}{t}$ is the Hubble constant.

Kinetic energy

$$\sim \tfrac{1}{2}m(HR)^2$$

Gravitational energy

$$\frac{G(\frac{4\pi}{3}R^3\rho)m}{R} = \frac{4\pi}{3}GmR^2\rho$$

$$\downarrow$$

$$\rho = \frac{3}{8\pi}\frac{H^2}{G} \sim \frac{1}{Gt^2}$$

Ultimate Computational Power of the Universe

$$\rho = \frac{3}{8\pi}\frac{H^2}{G} \sim \frac{1}{Gt^2}$$

- Plug and chug:
  - Total number of operations performed by the universe is $\sim \frac{(ct)^3 \rho c^2}{\hbar} = \frac{\rho c^5 t^4}{\hbar} \sim \frac{1}{Gt^2}\frac{c^5 t^4}{\hbar} = \frac{t^2}{t_p^2}$
  - This is ~ 10120 operations that the universe can compute over the course of its life time
  - Shine some light: number of operations is a dimensionless number
    - We are expecting $f(\frac{t}{\tau})$
    - Horizon of the universe expand in square term, so we are of course expecting $\frac{t^2}{t_p^2}$

And that's it!

Outline

- Origin and Motivation for Entropy
- Entropy and Planck
- Entropy and Quantum Computing
- Computational Power of a Black Hole
- Ultimate Computational Power of the Universe

Further Reading

[1] Jacob D. Bekenstein, Generalized second law of thermodynamics in black-hole physics, Phys. Rev. D 9 (1974), 3292–3300.

[2] Raphael Bousso, The holographic principle, Reviews of Modern Physics 74 (2002), no. 3, 825–874.

[3] E. G. D. Cohen, Entropy, probability and dynamics, 2008.

[4] E. Fermi, Thermodynamics, Dover Publications, 2017.

[5] R. Landauer, Irreversibility and heat generation in the computing pro- cess, IBM Journal of Research and Development 5 (1961), 183 – 191.

[6] S. Lloyd, Ultimate physical limits to computation, Nature 406 (2000), no. 6799, 1047–1054.

[7] S. Lloyd, Computational capacity of the universe, Physical Review Let- ters 88 (2002), no. 23.

[8] Don N. Page, Particle emission rates from a black hole: Massless par- ticles from an uncharged, nonrotating hole, Phys. Rev. D 13 (1976), 198–206.

[9] M. Planck, Eight lectures on theoretical physics delivered at columbia university in 1909, Martino Fine Books, 2019.

[10] L. Susskind and A. Friedman, Quantum mechanics: The theoretical min- imum, Penguin Books Limited, 2014.

[11] H. von Helmholtz, Ueber die physikalische bedeutung des princips der kleinsten wirkung, Journal fu¨r die reine und angewandte Mathematik (1887).